The experimental background of quantum theory

The black body radiation

The photoelectric effect

The dual nature of light. Matter waves

The black body radiation

Heat transport between two bodies can occur not only in a medium, but also by radiation. This radiation is of electromagnetic nature.

When attempting to describe the electromagnetic radiation emitted by a body, the quantitative description is greatly simplified, if we work with an idealized heated solid, called a cavity radiator.

A cavity radiator consists of a block of material, having an internal cavity connected to the outside of the block by a small hole. When the block is heated to a certain temperature, the radiation emerging from the hole is called cavity radiation, or black body radiation. The reason for the latter name is that the hole behaves as a perfect absorber: light that enters this hole is trapped within the cavity. The nature of the black body radiation depends only on the temperature of the cavity walls, and not at all on the shape or size of the cavity, or the material forming the cavity walls.
**Observations**

1) *Spectral distribution.* The spectrum of the radiation varies with temperature as shown in Figure 1.

![Figure 1](image.png)

**Figure 1.** The spectrum of black body radiation.

2) *Stefan-Boltzmann law.* The total energy emitted, that is proportional with the area under the curve, varies with the temperature as:

\[ E = \sigma \cdot T^4 \]

where \( \sigma \) is a constant.

For example, if the temperature rises from 2000 K to 4000 K, the energy increases 16-fold.

3) *Wien's law.* The wavelength at the maximum intensity (\( \lambda_m \)) is shifted towards lower wavelengths as temperature increases:

\[ \lambda_m \cdot T = \text{const.} \]

The formula describes the experience that, the light emitted by a glowing metal is red when it is hot, yellow at high, and white at very high temperatures.
**Theoretical explanation**

Theories based on classical physics had only limited success in explaining the experimental results.

The most important trials belong to Wilhelm Wien, and Lord Rayleigh and Sir James Jeans. The Wien-law fits well only for relatively short wavelengths, while the Rayleigh-Jeans formula fits well for higher wavelengths, however, they fail miserably outside these ranges. For example the Rayleigh-Jeans formula predicts infinite intensities when wavelength approaches zero. One formula fits the experimental results just where the other formula fails.

The solution was given by Max Planck, who created an empirical formula by interpolation between the two known formulas. His formula described perfectly the results, however, it had no theoretical basis for that moment.

When Planck started to elaborate the theoretical background, he assumed, that the atoms of the cavity walls behave like tiny electromagnetic oscillators, each having a characteristic frequency. The oscillators emit electromagnetic energy into the cavity and absorb electromagnetic energy from it.

To be able to explain his formula Planck was led to make two radical assumptions about the atomic oscillators:

1) An oscillator cannot have just any value of energy, but only values given by:

\[ E = n hf \]

where \( f \) is the oscillator frequency, \( h \) is a constant (now called the Planck constant), and \( n \) is a positive integer.

2) The oscillators do not radiate energy continuously, but only in "jumps", or quanta. The energy of one quanta is:

\[ \Delta E = hf \]

These assumptions were so radical, that even Planck resisted accepting them for many years.

For now it has been proved, that energy is quantized in all processes. However, we do not experience the discontinuity in our everyday life, because the Planck constant has a very low value, namely:

\[ h = 6.6 \times 10^{-34} \text{ joule.second.} \]
Planck described his theory to the Berlin Physical Society on December 14, 1900. A new epoch in physics started on that date.

Planck applied the quantization of energy only to the atomic oscillators, not to the radiation. It was Albert Einstein (1879-1955), who successfully applied the theory to the electromagnetic radiation, and named the quanta of light photons.

Based on this theory he was able to explain the experimental results of photoelectric effect, for which he was awarded with the Nobel-prize in 1921.
**The photoelectric effect**

The photoelectric effect is one of the most important phenomena proving the particle nature of light.

In the experiment monochromatic light of suitably high frequency falls on a metal surface, and ejects electrons (called photoelectrons) from it (Figure 2).

![Figure 2. The apparatus used to study the photoelectric effect.](image)

The electrons may be attracted to or rejected from the anode by applying a potential difference \((V)\). If they reach the anode, they form a photoelectric current \((I)\).

The two key parameters followed in the experiment are the number and the kinetic energy of the photoelectrons.

The number of photoelectrons reaching the anode \((N)\) is proportional to the photoelectric current \((I)\), because:

\[
I = \frac{Q}{t} = \frac{N \cdot e}{t},
\]

where \(Q\) is the charge, \(t\) is the time, \(e\) is the electron charge.

The kinetic energy of the individual electrons \((E_k)\) is proportional to the stopping potential \(V_0\), at which the photoelectric current drops to zero. In this case the electric field opposing the motion of the electrons prevents them from reaching the anode. The stopping potential multiplied by the electron charge measures the kinetic energy of one electron:

\[
E_k = e \cdot V_0.
\]
**Observations**

The current \( (I) \) depends on the potential difference as shown in Figure 3:

![Figure 3. Dependence of current on potential difference taken for two intensities of the incident light.]

As seen from the graph, the number of photoelectrons \( (\sim I) \) depends only on the intensity of the incident light, while the \( V_0 \), and thus the kinetic energy of the electrons does not depend at all on the light intensity.

The classical wave theory, which considered the light a continuous flow of energy, was unable to explain these results:

<table>
<thead>
<tr>
<th>Predictions of the wave theory</th>
<th>Experimental results</th>
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<tr>
<td>as the intensity of light increases, ( E_k ) should increase</td>
<td>as light intensity increases, the number of photoelectrons increases</td>
</tr>
<tr>
<td>the effect should occur for any frequency of light</td>
<td>it occurs only for frequencies higher than ( f_0 ) (cutoff frequency)</td>
</tr>
<tr>
<td>the electrons should be ejected with time delay, because they would slowly receive energy from the light</td>
<td>the effect is instantaneous</td>
</tr>
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</table>
**Theoretical explanation - Einstein's photon theory**

In 1905 Einstein made a remarkable assumption about the nature of light, using which he could explain the photoelectric effect: Under some circumstances light behaves as energy was concentrated into localized bundles, later called *photons*. The energy of a single photon is

\[ E = hf \]

where \( f \) is the frequency of the light, and \( h \) is the Planck constant.

This view, that light behaves a stream of particles, is in sharp contrast with the wave theory!

By applying Einstein's photon concept to the photoelectric effect, we can write:

\[ hf = \Phi + E_k. \]

The equation describes that a photon carries the energy \( hf \), which is absorbed by a single electron. Part of this (\( \Phi \), work function) is used in causing the electron to escape from the surface. The excess energy (\( hf - \Phi \)) becomes the electron kinetic energy.

The photon theory explains the experimental observations the following way:

1) If we increase the light intensity, we increase the number of incident photons, thus the number of photoelectrons increases.

2) By increasing the frequency of light we increase the energy of the individual photons, thus the kinetic energy of the electrons increases (see equation).

3) If the energy of a photon is smaller than the work function, electrons are not ejected at all, because the absorbed energy is not enough for an electron to leave the surface. In the limit case the photon energy is just enough to eject an electron, which then carries no extra kinetic energy:

\[ hf_0 = \Phi \]

where \( f_0 \) is the cutoff frequency.

4) The effect is instantaneous, because the energy is supplied in concentrated bundles.

For the 26-year old Einstein 1905 was a banner year. During that year he published, in sequence and in the same prestigious journal: (1) his theory of photons, (2) a theory of Brownian motion, (3) his special theory of relativity, and (4) the germ of his now-famous \( E=mc^2 \) formula. During all this time he worked as a minor official in the Swiss patent office, working on physics privately and in his spare time!
The dual nature of light. Matter waves

The dual nature of light

The particle nature of light has finally been accepted for the description of the photoelectric effect. However, based on other phenomena, the wave nature of light must also be valid.

Now the two views are considered as complementary aspects. Some phenomena can be understood by assuming that light is a wave, others by assuming that it behaves as a stream of particles. Examples:

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<th>Particle nature</th>
<th>Wave nature</th>
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<td>interference</td>
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<td>photoelectric effect</td>
<td>diffraction</td>
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<td>Compton effect</td>
<td>polarization</td>
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Matter waves

In 1924 Louis de Broglie reasoned that if light has a dual, wave-particle nature, perhaps matter has also. Since matter was at that time regarded as being composed of particles, de Broglie's reasoning suggested that it might be fruitful to search for a wave-like behaviour of matter.

De Broglie not only suggested the existence of matter waves, but also predicted what their expected wavelength would be. His famous formula allows calculation of this wavelength for a particle with the momentum \( p \) (\( p = \text{mass} \times \text{velocity} \)):

\[
\lambda = \frac{h}{p},
\]

where \( h \) is the Planck constant. The formula establishes a connection between the particle and wave aspects. The greater the momentum, the smaller the wavelength.

De Broglie's idea seemed to be only an abstract theory, till in 1927 Clinton Davission and Lester Germer in the USA, and George P. Thomson in Scotland proved experimentally that electrons may have wave nature. When an electron beam was allowed to fall on a single crystal of nickel, diffraction maxima were observed in the "reflected" beam. Moreover, the wavelength determined experimentally from the Bragg relationship was in an excellent agreement with that predicted by de Broglie's formula. This was a strong evidence that electrons are wave-like in some circumstances.
A crystal was necessary to observe the diffraction of electrons, because detectable diffraction occurs only if the distance between the centers of the diffracting grid (atomic planes in this case) are comparable to the wavelength. It turned out, that electron beams had similar wavelengths as X-rays used in crystallographic studies.

An application of the theory of matter waves was electron microscopy. The resolution of light microscopes is limited by the wavelength of light. The smallest resolvable distance for an electron microscope is much smaller than that for a light microscope, because the wavelength of the electron beam used to "illuminate" the sample is much smaller than the wavelength of light.

The matter-wave concept can also be applied to an electron in an atom, and it may help understand the Bohr quantization assumption. The orbiting electron in this case can be regarded as a wave, for which the wavelength must fit around the orbit an integral number of times.

Now we know, that not only electrons, but all other particles, charged or uncharged show wave-like characteristics.

According to complementarity principle formulated by Niels Bohr the wave-particle natures of either matter or radiation complement each other. It is not possibile to demonstrate the exclusive wave or particle nature of either matter or radiation. Both models are required.