Fluid flow
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Fluids
- Fluids – continually deform (flow) under shear stress
  - Liquid
  - Gas
  - Plasma
- Liquids
  - Short-range crystalline organisation that always reforms
  - Constant volume, incompressible
  - Moderate resistance to deformation: fit the shape of solid surrounding (container) or force field
  - No preferred direction

Fluid mechanics
- Hydrostatics (resting fluids)
- Hydrodynamics (moving fluids)
  - Ideal fluids (no internal friction)
  - Real (viscous) fluids
    - Newtonian fluids
    - Non-Newtonian fluids
- Flow
  - Laminar flow
  - Turbulent flow
  - Stationary flow (equal amounts of mass or volume through the cross section in a unit of time)
  - Changing in time

Pascal’s law
\[ p = \frac{F}{A} \]
\[ W = p \cdot \Delta V \]
Fluids are incompressible:
\[ A_1 \cdot d_1 = A_2 \cdot d_2 \]
\[ p_1 \cdot A_1 \cdot d_1 = W_1 = W_2 = p_2 \cdot A_2 \cdot d_2 \]
\[ \frac{p_1}{p_2} = \frac{A_2}{A_1} = \frac{A_1}{A_2} = F_1 < F_2 \]
Pressure exerted anywhere on a fluid in a confined space is transmitted to an equal extent in all directions.

Hydrostatic pressure
- In a gravitation field, pressure is proportional to height (depth) because of the weight of the fluid column.
\[ F = \dot{G} = m \cdot g \]
\[ p = \frac{F}{A} = \frac{m \cdot g}{A} = \rho \cdot V \cdot g \frac{A}{A} = \rho \cdot A \cdot g = \rho \cdot h \cdot g \]

- If also atmospheric pressure is considered:
  \[ p = p_{\text{atm}} + \rho \cdot h \cdot g \]

1 mmHg (1 Torr) = 133.3 Pa
1 atm = 101 325 Pa = 101 kPa
- For interconnected fluids in equilibrium:
  \[ p_1 = p_2 \quad \rho_1 \cdot h_1 \cdot g = \rho_2 \cdot h_2 \cdot g \quad \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \]
- Independent of the fluid’s shape
Law of Archimedes

- Objects immersed in fluid lose weight

\[ F = p \cdot A = \rho \cdot g \cdot h \]

Law of continuity

- Fluids are incompressible
- In \( \Delta t \) time the flow volume through any cross-section is the same:

\[ \dot{V} = \frac{\Delta V}{\Delta t} \]

Bernoulli's law

- Potential energy:

\[ \Delta (m \cdot g \cdot h) = m \cdot g \cdot h_2 - m \cdot g \cdot h_1 \]

\[ p_1 \cdot \Delta V + \frac{1}{2} \rho \cdot v_1^2 + m \cdot g \cdot h_1 = p_2 \cdot \Delta V + m \cdot g \cdot h_2 + \frac{1}{2} \rho \cdot v_2^2 \]

Flow

- Movement of fluids in one direction
- Driving force is the pressure difference
- Foundational axioms: conservation laws (mass, energy, momentum)
- Continuum assumption
  - Fluids are continuous matter rather than made up of molecules.
  - Physical properties are well-defined at infinitesimal points, and vary continuously from one point to another.
- Intensity of current or volumetric flow rate

\[ \dot{V} = \frac{\Delta V}{\Delta t} \]

Bernoulli's law in real fluids

- Laminar flow in real fluids
- For rigid tubes, ideal fluids and stationary flow: (conservation of mass)

\[ I = \dot{A} \cdot \dot{y} \]

Laminar flow in real fluids

- Constant cross-section
- Pressure decreasing in the direction of flow
- Proportional to distance
- Fluid resists to the moving effect (flow) with a force
- An internal friction between imaginary fluid layers sliding past each other — decreasing displacement profile

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Newton's law

\[ \text{shear stress} = \frac{F}{A} \]

\[ \text{strain rate} = \frac{\Delta \varepsilon}{\Delta t} \]

\[ \text{viscosity} = \frac{\text{shear stress}}{\text{strain rate}} \]

\[ \frac{N}{m^2} \times \frac{m}{s} = \frac{N}{s} \]

\[ F = \eta \frac{\Delta v}{\Delta x} \]

Isaac Newton (1643-1727, ENG)

- Viscosity depends on matter, temperature, concentration, pressure
- Ideal fluid: zero viscosity (e.g., certain liquid He species)
- Newtonian fluid (e.g., water):
  - Viscosity = shear stress
- Non-Newtonian fluid (e.g., blood):
  - Viscosity not proportional only to shear stress
  - It depends on flow velocity
- Viscosity of gases increases at higher temperatures, that of fluids decreases

### Hagen-Poiseuille law

Force because of pressure difference = frictional force (laminar, stationary flow, rigid tube)

\[ \Delta p \cdot A = (p_1 - p_2) \cdot A = F = -\eta \cdot \frac{\Delta v}{\Delta t} \]

\[ \Delta p = \frac{F}{A} \]

\[ \frac{p_1 - p_2}{L} = \frac{\rho \cdot \Delta v}{R^4} \]

Gotthilf Hagen (1797-1884, GER)
Jean Poiseuille (1797-1869, FRA)

- Laminar flow
  - Low speed
  - No swirling
  - On smooth surface
- Turbulent flow
  - High speed for viscosity
  - Swirling, no "layers"
  - On rough surface (blood vessels)
- Reynolds number
  \[ R = \frac{\rho \cdot u \cdot L}{\eta} \]

Osborne Reynolds (1842-1912, IRE)

Critical velocity for smooth tubes

\[ R_{crit} \approx 1160 \]

- Hydrostatic resistance

- A medium (gas or liquid) exerts an opposite force to the direction of the movement on the objects moving in it:

\[ F = \frac{1}{2} \rho \cdot A_r \cdot v^2 \]

- Streamline bodies (small k):
  - Flow layers unite behind the object, low resistance
- Non-streamline bodies (great k):
  - The medium flows rapidly behind the object
  - Low pressure → suction effect → great counter-force

- Terminal (settling) velocity

\[ \frac{F_{sett}}{F} = \frac{\Delta p}{\rho \cdot g \cdot r^2} \]

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George Stokes (1819-1903, IRE)

- Movement of bodies in real fluids
- Frictional force on moving sphaerical objects:

\[ F_{friction} = \frac{1}{2} \rho \cdot A_r \cdot v^2 \]

- Terminal (settling) velocity

\[ v = \frac{2}{9} \left( \frac{\rho_{air} - \rho_{water}}{\rho_{water}} \right) g \cdot r^2 \]

2015.11.12.

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Summary

- Pascal's law: transmission of pressure
- Continuity equation: relationship of surface and flow velocity
  \[ \frac{V}{V} = \text{constant} \]
- Bernoulli's law: relationship of pressure and velocity
  \[ \frac{p}{\rho} + \frac{V^2}{2g} + h = \text{constant} \]
- Newton's law: internal friction
  \[ F = \rho \cdot A \cdot \frac{dV}{dt} \]
- Hagen-Poiseuille law: flow of real fluids
  \[ Q = \frac{\pi r^4}{8\eta L} \]
- Reynolds-number: critical velocity of the turbulent flow
  \[ \frac{V_c}{v} \]
- Stokes' law: objects moving in a medium

THANK YOU FOR ATTENTION!

Torrerelli's law

Specific case of Bernoulli's law

- At the top and at the opening: \( p = p_{\text{atm}} \)
- At the top: \( v = 0 \) at the opening: \( h = 0 \)
  \[ \rho \cdot g \cdot h + p_{\text{atm}} = \rho \cdot \frac{V^2}{2g} + p_{\text{atm}} \]
  \[ v = \sqrt{2 \cdot g \cdot h} \]

Evangelista Torricelli (1608-1647, ITA)

Venturi effect

- Flow through a constriction
- Gain in kinetic energy
- Loss in pressure
- Parfume spray, chimney

Giovanni Venturi (1746-1822, ITA)