BIOMECHANICS 4
Origins and consequences of forces in biological systems

MOLECULAR MECHANISMS OF BIOLOGICAL MOVEMENT
- MUSCLE MECHANICS
### Muscle contraction – muscle response

<table>
<thead>
<tr>
<th>STIMULUS</th>
<th>MUSCLE RESPONSE</th>
<th>CONTRACTION PATTERN</th>
</tr>
</thead>
<tbody>
<tr>
<td>single</td>
<td>TWITCH</td>
<td><img src="image" alt="Twist" /></td>
</tr>
<tr>
<td></td>
<td>a single contraction &amp; relaxation event</td>
<td></td>
</tr>
<tr>
<td>paired</td>
<td>WAVE SUMMATION</td>
<td><img src="image" alt="Summation" /></td>
</tr>
<tr>
<td></td>
<td>S1: partial contraction on the arrival of S2: S1+S2</td>
<td></td>
</tr>
<tr>
<td>more low frequency</td>
<td>&quot;rare&quot; UNFUSED, INCOMPLETE TETANUS</td>
<td><img src="image" alt="Incomplete Tetanus" /></td>
</tr>
<tr>
<td>&quot;rare&quot;</td>
<td>partial relaxations</td>
<td></td>
</tr>
<tr>
<td>more high frequency</td>
<td>&quot;frequent&quot; COMPLETE TETANUS</td>
<td><img src="image" alt="Complete Tetanus" /></td>
</tr>
<tr>
<td>&quot;frequent&quot;</td>
<td>continuous contraction without relaxation (abnormal case: spasm, pathology: Clostridium tetani)</td>
<td></td>
</tr>
</tbody>
</table>

### Cross-bridge cycle – force generation

**Movement of the myosin II head in one crossbridge cycle:**

\[ d_{crossbridge} \approx 10 \text{ nN} = 10 \times 10^{-9} \text{m} \]

**Force generated in one crossbridge cycle:**

\[ F_{crossbridge} \approx 2 \text{ pN} = 2 \times 10^{-12} \text{N} \]

**Number of cross-bridges:** \( N \)

- 1 thick filament \( \sim 200 \text{ myosin} \) \( \rightarrow F \sim 2 \times 200 \text{ pN} = 400 \text{ pN} \)
- 1 myofibril \( \sim 10^4 \sim 10^5 \text{sarcomere} \)
- 1 fibre \( \sim 2000 \text{ myofibril} \)
- 1 muscle \( \sim 10^2 \sim 10^5 \text{ fibre} \)

\[ F \sim 2 \text{ pN} \times 200 \times 10^2 \times 2000 \times 10^5 = 8000 \text{ N} \sim 800 \text{ kg} \]

**Total force:** \( F_{total} = N \cdot F_{cross-bridge} \)

The number of acto-myosin crossbridges depends on:

- overlap between thin and thick filaments
- Frank-Starling law
- ATPase activity of myosin II: \( V_{ATP} \)

**Crossbridge cycle time:** \( t = \frac{1}{V_{ATP}} \)
Frank-Starling’s law: Increasing preload (tension of muscle fibers) leads to increase in stroke volume and the contraction force.

**Force – sarcomere length**

![Diagram of sarcomere length vs. tension](image)

**Force – velocity, power – velocity**

![Diagram of force and power vs. velocity](image)
**Torque**

- rotational equivalent of force (τ, tau)
- measures the **turning/twisting effectiveness** (changing the angular velocity over a period of time) **of a force**

\[ \vec{\tau} = \vec{r} \times \vec{F} \ [N m] \]

\[ \tau = rF \sin \alpha \]

- \( r \) = distance from the pivot point to the point where the force is applied
- \( F \) = magnitude of the force
- \( \alpha \) = angle between the force vector and the vector directed from the point of application to the pivot point

\( rF \sin \alpha \) = **lever arm** (perpendicular distance between the pivot and the force)

- counterclockwise rotation = positive torque
- clockwise rotation = negative torque

\[ \tau = rF \sin \theta \]

\[ F = 10 \text{ N} \]
\[ \sin \alpha = \sin 90^\circ = 1 \]
\[ r = 1 \text{ m} \]
\[ \tau = 10 \text{ Nm} \]

\[ F = 5 \text{ N} \]
\[ \sin \alpha = \sin 90^\circ = 1 \]
\[ r = 1 \text{ m} \]
\[ \tau = 5 \text{ Nm} \]

\[ F = 10 \text{ N} \]
\[ \sin \alpha = \sin 90^\circ = 1 \]
\[ r = 0.5 \text{ m} \]
\[ \tau = 5 \text{ Nm} \]
**Equilibrium**

**EQUILIBRIUM**

1st condition for equilibrium:
\[ \vec{F}_{\text{net}} = 0 \]

2nd condition for equilibrium:
\[ \vec{r}_{\text{net}} = 0 \]

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**Simple machines**

„Give me a lever long enough and a place to stand and i will move the earth.„

Aristoteles (384 – 322 BC)
Simple machines

1\textsuperscript{st} condition for equilibrium: \( \vec{F}_{\text{net}} = 0 \)
\( \vec{F}_{\text{input}} + \vec{F}_{\text{output}} + \vec{F}_{\text{pivot}} = 0 \)

2\textsuperscript{nd} condition for equilibrium: \( \vec{r}_{\text{net}} = 0 \)
\( \vec{r}_{\text{net}} = \vec{r}_{\text{input}} + \vec{r}_{\text{output}} + \vec{r}_{\text{pivot}} = 0 \)
\( F_{\text{input}} \cdot d_{\text{input}} = F_{\text{output}} \cdot d_{\text{output}} \)

\[ \begin{array}{c|c|c|c}
F_{\text{input}} & F_{\text{output}} & \text{input} & \text{output} \\
100 N \sim 10 \text{ kg} & 100 N \sim 10 \text{ kg} & 1 \text{ m} & 1 \text{ m} \\
50 N \sim 5 \text{ kg} & 100 N \sim 10 \text{ kg} & 2 \text{ m} & 1 \text{ m} \\
200 N \sim 20 \text{ kg} & 100 N \sim 10 \text{ kg} & 0.5 \text{ m} & 1 \text{ m} \\
\end{array} \]

\[ \begin{array}{c}
F_{\text{input}} \cdot d_{\text{input}} = F_{\text{output}} \cdot d_{\text{output}} \\
F_{\text{input}} = F_{\text{output}} \cdot \frac{d_{\text{output}}}{d_{\text{input}}} \\
\end{array} \]

\textbf{relation between} \( d_{\text{output}} \) \textbf{and} \( d_{\text{input}} \) \textbf{\Rightarrow relation between} \( F_{\text{output}} \) \textbf{and} \( F_{\text{input}} \)

- \( d_{\text{output}} > d_{\text{input}} \rightarrow F_{\text{input}} > F_{\text{output}} \): MECHANICAL DISADVANTAGE ☹
- \( d_{\text{output}} < d_{\text{input}} \rightarrow F_{\text{input}} < F_{\text{output}} \): MECHANICAL ADVANTAGE ☺
https://www.youtube.com/watch?v=d1w5OlJzmI

Type 1 lever

head

skull-1st vertebrae

neck muscles

F_{output}

F_{input}

Movement completed

Resistance

Fulcrum

Applied force

Type 2 lever

- Body weight
- Leg muscles
- Joints

Type 3 lever

- Biceps
- Elbow joint
- Forearm

Movement completed
Even when the head is held erect its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class.

Calculate the force exerted by these muscles.

What is the force exerted by the pivot on the head?

The ratio of the force exerted by the muscles to the total weight held by muscles?

MECHANICAL ADVANTAGE

Force exerted on the pivot (atlanto-occipital joint) of the head:

The 75-kg man stands on his toes by exerting an upward force. Calculate the force exerted by the muscles of the back of the leg.
The figure shows the forearm of a person holding a book. The biceps exert a force to support the weight of the forearm and the book. The triceps are assumed to be relaxed.

Calculate the force the biceps muscle must exert to hold the forearm and its load.

\[
F_{\text{input}} = \frac{38\, \text{cm} \times 4\, \text{kg} \times 9.81\, \text{m/s}^2 + 16\, \text{cm} \times 2.5\, \text{kg} \times 9.81\, \text{m/s}^2}{4\, \text{cm}} = 470\, \text{N}\uparrow
\]

Weight of the book and the forearm:

\[
F_{\text{book+forearm}} = 4\, \text{kg} \times 9.81\, \text{m/s}^2 + 2.5\, \text{kg} \times 9.81\, \text{m/s}^2 = 63.7\, \text{N}\downarrow
\]

The ratio of the force exerted by the biceps to the total weight held by the biceps:

\[
\frac{F_{\text{muscle}}}{F_{\text{book+forearm}}} = \frac{470\, \text{N}}{63.7\, \text{N}} = 7.3!!
\]

**MECHANICAL DISADVANTAGE**