FLUID FLOW

Textbooks and resources

Medical Biophysics (Damjanovich S, Fidy J, Szőllősi J editors) Medicina Press, 2009

An Introduction to Biophysics (with medical orientation) (Györgyi Rontó and Imre Tarján editors) Akadémiai Kiadó, Budapest

Main topics:

1. Hydrostatics
2. Hydrodynamics
3. Medical references
4. Real fluids

Liquids (viscous liquids) take on the shape of the container used for storage, keep their volume (incompressible) and their particles constantly change their place in any suitable manner.

Pressure

Pressure is the fraction of force and the area the force is applied on (p=F/A). Its unit is N/m=Pa, but we use bar, atm and mmHg=torr too. It is a scalar quantity, therefore it has no direction.

Hydrostatics

Physics of non-flowing liquid.

Pascal’s law: Pressure exerted anywhere on a fluid in a confined space is transmitted to an equal extent in all directions.

Hydrostatic pressure:

\[ p = \frac{mg}{A} = \frac{\rho Ah}{A} = \rho gh. \]

Hgmm: a mercury column in equilibrium with the air pressure is 760 mm high, therefore the atmospheric pressure is 760 Hgmm.
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\[ p = \rho_{mercury} gh = 13595 \frac{kg}{m^3} \cdot 0.760 m \cdot 9.806 \frac{m}{s^2} = 101317.5 \frac{kg}{ms^2} = 1.01 \cdot 10^5 \text{ Pa} \] is the atmospheric pressure in Si units.

1 bar = \(10^5\) Pa
1 atm = \(1.013 \cdot 10^5\) Pa
760 Torr = 1 atm

Hydrodynamics

Physics of flowing fluids.

Driving force: pressure difference. Conserved quantities: material, energy and momentum.

**Continuity equation**

Volumetric flowrate: \((Q)\) is the volume of fluid which passes per unit time. \((m^3/s\) or liter/s). Eg. In the aorta blood flows with 5 liter/min.

Stationary flow: the volumetric flowrate is constant.

\[ Q = \frac{V}{t} = \frac{A d}{t} = A \cdot v \]

Tubes with changing cross-sections: \(A_1 \cdot v_1 = A_2 \cdot v_2\)

Eg. water drop, vessels

**Bernoulli’s law**

In flowing fluids the sum of static \((p)\), dynamic \((1/2\rho v^2)\) and hydrostatic \((\rho gh)\) pressure is constant.

\[ p + \frac{1}{2} \rho \cdot v^2 + \rho \cdot g \cdot h = \text{constant} \]

Static pressure (pushing the walls) decreases if the velocity of the flow increases. Bernoulli’s law is a form of energy conservation from which it can be derived.

**Medical reference**

Aneurism
Formation of aneurism: 1) a weakened part of the vessel dilates, 2) at the dilated part the velocity of blood decreases (continuity equation), 3) the dynamic pressure decreases and the static pressure pushing the walls of the vessel will increase (Bernoulli’s law), 4) which will further dilate the vessel, the effect intensifies (positive feedback).

Venturi effect

The higher velocity of flowing fluids or gases coming through a narrowing tube results in static pressure decrease. This mechanism allows the air intake in an oxygen mask. Holes of various sizes on the walls of the mask regulate the variety of air-oxygen ratio.

Real fluids

Viscosity

Inner resistance, resistance to flow.

Ideal fluids viscosity is zero, only real fluids have viscosity.

Interpretation: (Newton’s law): force (F) by which two plates— with liquid in between— will slide on each other by a certain velocity gradient (Δv/Δy).

\[ F = \eta \cdot A \frac{\Delta v}{\Delta y} \]

Liquids for which this law is valid are called Newtonian fluids. Fluids found in joints (knee) are not Newtonian fluids. They are called synovial fluids and their viscosity decreases with pressure, working more efficient than greases.

Viscosity depends on material, temperature and pressure.
Velocity gradient: Because of Viscosity - that is the inner resistance between the layers of the fluid and the fluids and the walls- the velocity of the fluid is at its maximum in the middle of the tube and decreases toward the walls. The velocity of the layer next to the wall is theoretically zero, and the profile shows a parabola shape.

Consequence: the red blood cells are concentrated along the axis of the vessels, because the pressure increases towards the walls of the vessels (Bernoulli’s law).

**Hagen-Poiseuille’s law**

Pronounce: hágen-poasoí.

Describes how the volumetric flowrate depends on the properties of the tube and the fluid:

\[ Q = \frac{\pi \Delta p r^4}{8 \eta l} \]

- Proportional to pressure difference of the two end of the tube (\(\Delta p\))
- Proportional to the fourth power of the tube’s (\(r^4\))
- Inversely proportional to the fluid’s viscosity (\(\eta\))
- Inversely proportional to the length of the tube (\(l\)).

It is an analogue of Ohm’s law:

Analogies:
- Volumetric flowrate – electric flowrate
- Pressure difference- potential
- Flowing resistance – electric resistance

Important consequence: If an organ needs double the blood, blood pressure should be doubled, but because of the fourth power of the radius, it is enough if the radius of the vessels is increased by 19
The body, therefore, can effectively control the volume of blood flow through the diameter of the blood vessels.

**Stokes’s law**

Force acting on a sphere inside flowing fluids:

\[ F = 6 \cdot \pi \cdot \eta \cdot r \cdot v \]

\( \eta \) is the viscosity, \( r \) is the radius of the sphere, \( v \) the velocity of the fluid.

(\( 6\pi \eta r \) expression is the shape factor, mentioned at centrifugation)

**Turbulent flow**

The law of Hagen-Poiseuille is only valid for laminar flow that is for parallel flowing layers, without mixing. Obstacles, sudden narrowing, or if speed exceeds a critical value the flow becomes turbulent. Energy absorbing eddies, chaotic whirls, and vortexes appear, which do not add to the volumetric flowrate moreover increase the resistance of the flow.

**Reynolds number**

A number without unit, which allows to estimate the critical velocity above the flow turns turbulent:

\[ R = \frac{v \cdot r \cdot \rho}{\eta} \]

\( v \) is the effective (average) velocity if the flow, \( \rho \) is the density of the fluid, \( r \) is the radius of the tube, \( \eta \) is the viscosity.

If the calculated Reynolds number is above the critical Reynolds number (1160 for smooth-walled pipe) the flow is probably turbulent. Rearranging the equation provides the critical velocity above the flow becomes turbulent:

\[ v_{krit} = R_{krit} \frac{\eta}{\rho \cdot r} \]

The blood flow is laminar, turbulent flow only occurs in the aorta.